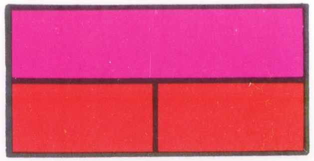
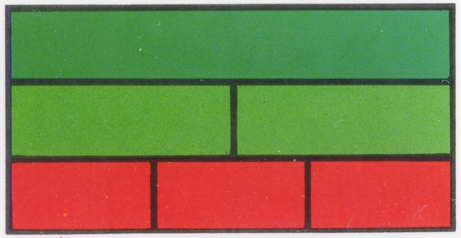


BOOKLET No. I

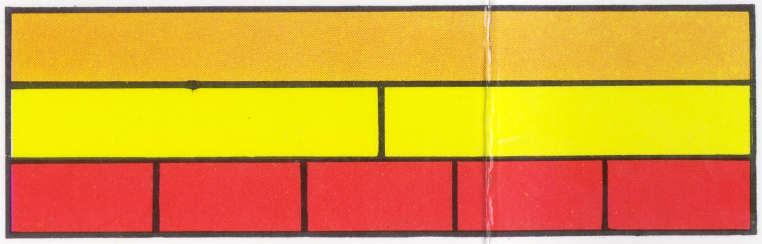


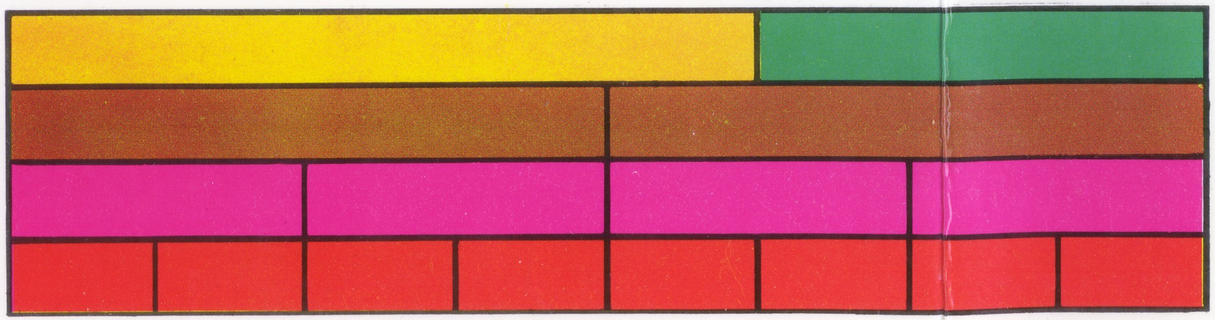
MAKING TRAINS
OF RODS OF
ONE COLOUR ONLY

—JUST FOR FUN

AND FOR STUDYING
PRODUCTS, FACTORS,
MULTIPLICATION FACTS

. . . and MORE





A BOOK OF IDEAS FOR EVERYONE WHO HAS CUISENAIRE RODS

This book belongs to...

After years of experimenting, Georges Cuisenaire, the inventor of the rods, eventually decided over a quarter of a century ago that the most pleasing and effective choice of colours for them were those you find in your set today. This series relating colours and lengths is protected by World Copyright. The contents of this Book of Ideas is based on the work and writings of Dr. C. Gattegno.

Further information on the use of your rods and other Cuisenaire materials may be obtained from

The Cuisenaire Company,

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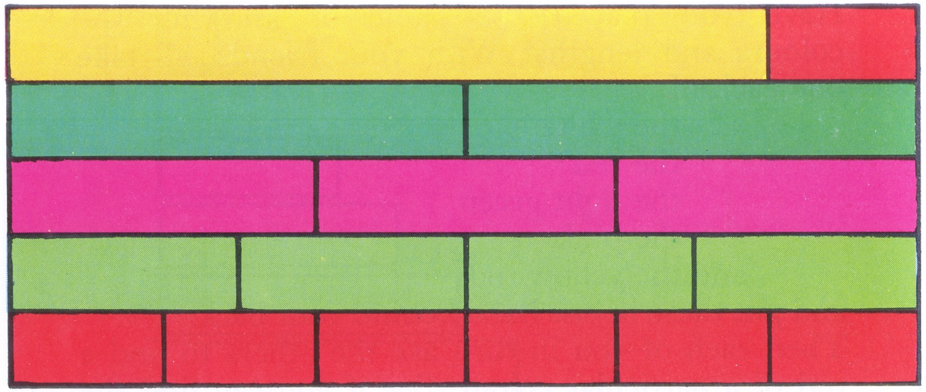


Now that you have your own set of Cuisenaire rods you can soon be expert at building, making patterns and creating rod pictures. If you already go to school then you have probably used them before now. You need not lose any as you always know they are all back in the box when they fill it exactly.

There is plenty you can find to do with the rods without starting on this book for some time. In fact there is so much to do altogether that you may well find them useful years from now.

One of the best ways of finding things out is to think about them on your own. Working on your own with the rods will help you to understand some things you will always find useful.

In this book there are some ideas for using your rods you may not have already thought of for yourself.

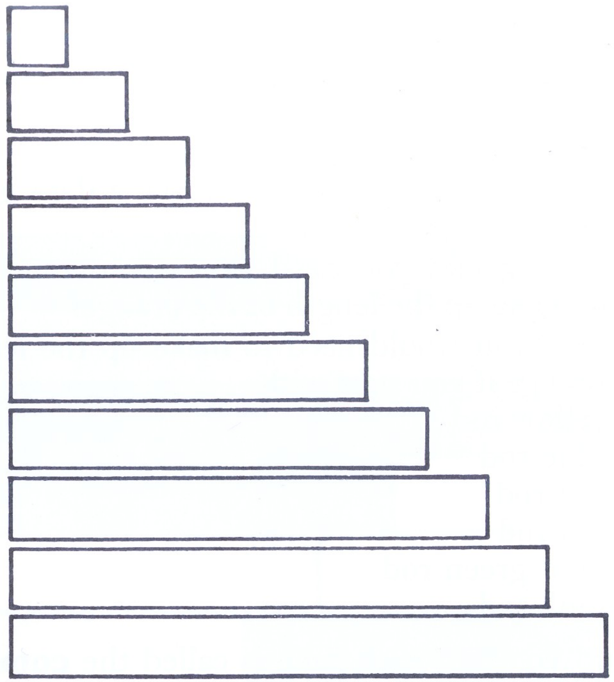
Have you played the game of knowing the colours and lengths? With your friends, all take one each of

the white
the red
the light green
the pink
and the yellow rods

and put these in one hand behind your back.
Then each of you in turn ask the others to find a rod of a particular colour. See if you can find the rod that is wanted, without looking, just by the feel of its length. Each time you find the right one, put it back and try again, this time to find another colour.
When you can do this easily it is time to use the longer rods as well. Try it with one of each colour. If you find it hard to hold all these, put them in a box lid or on a chair and, keeping your eyes shut, try finding the colours asked for by feeling for the lengths.

Smaller and bigger

Is the yellow rod the same size as the pink rod?
Is it bigger or smaller?
Is it bigger than the black rod?
Is it bigger or smaller than the red rod?
If you take a pink rod can you find another rod that is equal to it?
One that is smaller?
One that is bigger?
Another one that is bigger?



Staircases

Can you remember the order of the colours when you make a staircase with the rods? Put down the longest one, and then the next longest, and then one each of the others down to the smallest rod.
Look at them. If you first say the colours aloud from the white to the orange and then back again, you will soon be able to do it without looking, keeping your eyes closed and remembering what the staircase looks like. When you can do this easily, try saying the colours for every other rod in the staircase, still with your eyes closed, first going up the staircase, then coming down.

Complementaries

Take an orange rod and place a black rod beside it. Which rod would you need to put at the end of the black to make up the length of the orange?

Keep the orange rod, but now place a dark green rod next to it.

Which rod would you need to put with the dark green to make up the length of the orange?

Find what you would need to make up the length of the orange if you start with

- a yellow rod
- a blue rod
- a red rod
- a tan rod
- a light green rod
- a pink rod

The rod you find each time is called the **complement** in the orange of the one given to you.

Put a black rod and a light green end to end. Which rod is equivalent in length to this train?

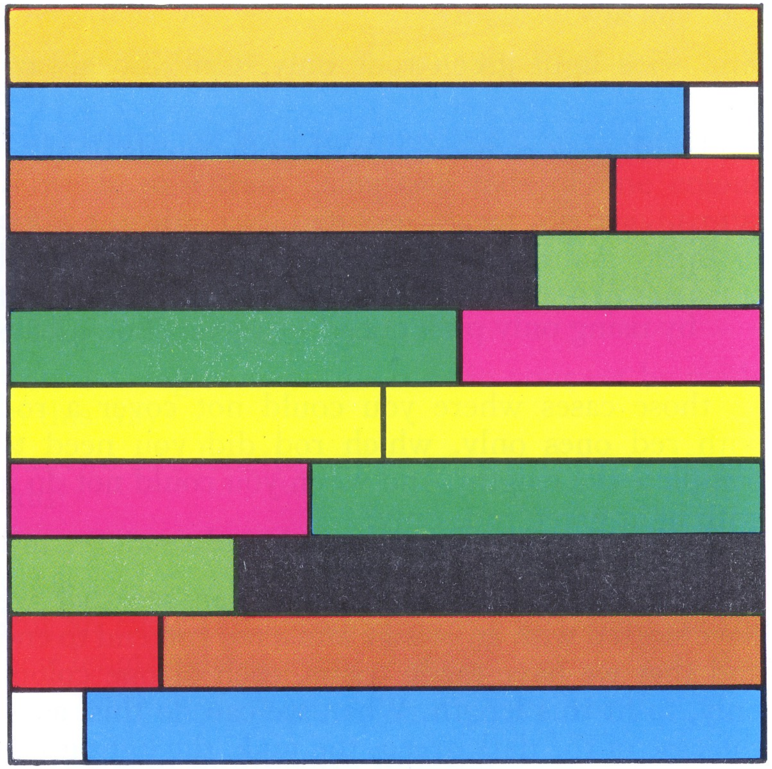
Find the rod equivalent to

- the pink and the dark green **train**;
- the red and the tan;
- the Blue and the white;
- the light green and the black.

Now take a Blue rod and place a yellow rod alongside it. The rod that would complete the length is the complement of the yellow in the Blue. Which rod is it?

Can you find the complements of the white, of the red, of the black, in the Blue?

The illustration below shows the complete table of complements in the orange rod.



Can you find the complete tables of complements in the tan rod, the black rod, the dark green rod, and so on down to the red?

Are these tables all the same size? Will the white rod have a table of complements too?

Trains of one colour

Try to form the length of any *one* rod by using red rods only.

Can you always do it?

And with light green rods only. Can you always do it?

And with pink rods only. With yellow rods only.

Which rods can be covered by using only red rods?

by using only light green ones?

by using only pink ones?

In those cases where you could not cover a rod with red ones only, which rod did you need to complete the length? And when you could not do it with light green rods only?

Look at the picture on *p.1*. You can see that we have started with an orange and a red rod end to end and that we have made trains, each of one colour only, to fit this length. When we can do this, as in the case of the dark green, the pink, the light green and red rods, we say that we have found the **factors** of the length, that is, found rods that fit in exactly a number of times. You can see that the orange rod is not a factor as it does not fit in exactly.

In the same way, try to find the factors of some lengths of your own choice.

Odd and even lengths

The rods whose lengths can be formed by trains of red rods only we shall call **even.**

The other rods we shall call **odd**.

By making trains, find all the odd lengths in your set.

Find which are the even ones.

If we form a train using two rods of one colour with a white rod between them, do we form a length that is odd or even? Form more trains in this way, using each time a pair of rods of the same colour and a white rod.

Can the lengths of all the rods in your set be made in this way?

Put two odd lengths end to end. Is the new length odd?

Put two even lengths end to end. Is the new length even?

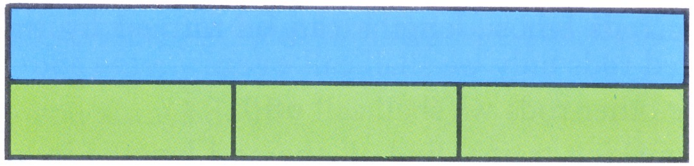
Put an even and an odd length end to end. Is the new length odd or even?

Prime and composite lengths

When a certain length has factors other than itself and the white rod, we call this length **composite.** Otherwise we call it **prime**. Is the length shown on page 16 (the orange and the black length) prime or composite?

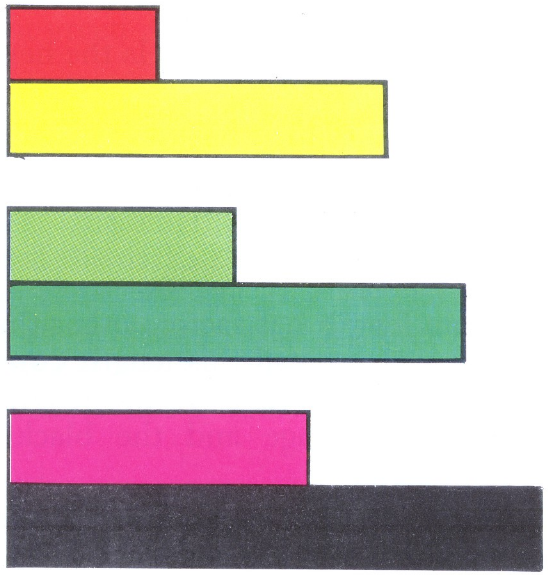
In your set of rods, find examples of prime and of composite lengths.

Addition and subtraction



This is an example of what we call **addition**: we have added the lengths of three light green rods to make a train equal in length to the Blue rod.

Look at the pictures below. Could you find which rod should be added to each of the patterns to make up the **difference** between each pair of rods? You can write your findings down as expressions in the way described on the opposite page.



The difference between each pair of rods is the same, for these are examples of what we call **equivalent subtractions**.

* * *

Instead of writing the names of the colours of the rods out in full, let us use the following short forms:

w for white	d for dark green
r for red	b for black
g for light green	t for tan
p for pink	B for Blue
y for yellow	o for orange

Also, when we wish to write that two rods are end to end, let us put the sign + and read it **plus**.

$w + r$ tells us that the white rod and the red rod are end to end.

Let us write − between two letters when we wish to show that we have two lengths and that we are looking for the difference between them. We shall read this sign **minus**.

$o - w$ tells us that we want to find the rod whose length is the difference between the lengths of the orange and the white rods.

You can find that B is the rod we want.

And let us put = whenever we want to say **equals**, or **is equivalent to**.

* * *

If you have understood what we have just done, you should be able to read the following **expressions:**

$w+r=g$ $\qquad$ $g=y-r$

Can you write down an expression for each pattern shown opposite?

Can you find the answers to the following:

$t-p=\square$

$t=r+\square$

$t-y=\square$

$y-(r+\square)=r$

$t-(\square+w)=g$

$w+g+g+\square=B$

$\square=g+d$

$o-2r=\square$

$B-b+r=\square$

$b-(w+\square+g)=r$

$d+b-(2p+w)=\square$

$g+r+y=\square$

$y+g-d=\square$

$t-\square+w=b$

$y-g-\square=w$

$\square-(b+r)=w$

$4w+2g=\square$

$B-(2r+p)=\square$

$d+(b-2g)=\square$

$\square=o-(2r+g)$

$3y-2p=\square$

$o+B-2t=\square$

$\square+r=y$

$y=g+\square$

$g+r=\square$

$w+r+\square=p$

$p-g=\square$

$y=r+w+\square$

$y=w+w+w+\square$

$w+r+\square+w=y$

$b-\square=r$

$\square-y=w$

$y-\square=g$

$\square=r+g$

$y-r=\square$

$\square+w=p$

$w+r+\square=y$

$d=\square+w$

$w+\square+y=b$

$g+p=\square$

$d-\square=p$

$d-g=\square$

You can find more examples of your own and write them here:

Working with numbers

Let us measure all the rods using the white rod as the **unit**.

How many white rods will be needed to make a train equivalent in length to a red rod?

How many will be needed for a light green rod? and for a pink rod?

Find how many rods will be needed to make trains equivalent in length to all the different rods in your set. The answers you get provide new names for the rods, which can be written down in the following way:

one for the white rod, or 1
two for the red, or 2
three for the light green, or 3
four for the pink, or 4
five for the yellow, or 5
six for the dark green, or 6
seven for the black, or 7
eight for the tan, or 8
nine for the Blue, or 9
ten for the orange, or 10

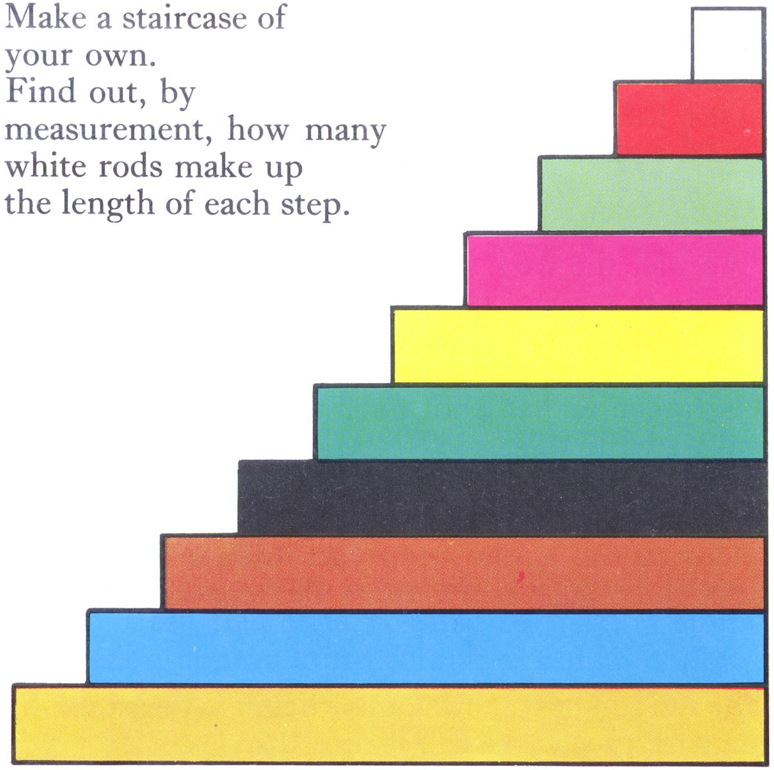
When we write expressions with these new names, we use **figures** to show the length of each rod when measured by the white rod.

What length would represent 11, and 12, and 23?

Complete these expressions:

$3+4=\square$	$11-3=\square$	$15-\square=7$
$\square=7+2$	$12-9=\square$	$\square+8=17$
$10+6=\square$	$\square=8+8$	$\square+7=11$

Make a staircase of your own.
Find out, by measurement, how many white rods make up the length of each step.



You can see that there is really very little that is new about working with numbers. What you are doing is the same; only the names given to the rods have changed. Now you can complete expressions similar to those you have met earlier but where instead of letters standing for colour names for the rods, numerical values are given them as measured by white rods.

Look at the expressions on pages 10 and 11 again. Try to complete them once more, this time using figures instead of letters.

Fractions

Look at the cover of this book. The pink rod is **twice** the length of the red, and the red we say is **half** the pink. We write this

$p=2\times r$ $\qquad r=\frac{1}{2}\times p$ where $\frac{1}{2}$ is read *half*.

Find half the dark green rod, and the orange, and the tan.

Four pink rods make up the lengths of the orange+dark green end to end. Each is **one quarter** of it and we write this

$o+d=4\times p$ $\qquad p=\frac{1}{4}\times(o+d)$

Each red rod is said to be **one fifth** of the orange since the orange is five times the length of the red.

$o=5\times r$ $\qquad r=\frac{1}{5}\times o$

The red rod has a new name when it is measured by the dark green. We call it **one third**:

$r=\frac{1}{3}\times d$ $\qquad d=3\times r$

Can you find a single rod equivalent to two thirds of the dark green. Could you find a single rod equivalent to $\frac{2}{5}$ of the orange, and to $\frac{2}{4}$ of the length orange plus dark green?

Can you complete the following expressions?

$\frac{2}{3}\times d+\frac{2}{5}\times o+\frac{1}{4}\times(o+d)+p=\square$

$\square=\frac{1}{2}\times p$ $\qquad r=\square\times t$ $\qquad t=\frac{1}{2}\times\square$

$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times(o+d)=\square$

You will have seen, from looking at the patterns on the cover, that the red is called in turn $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{8}$, each time being a fraction of a different rod. We can say that

$r=\frac{1}{2}\times p=\frac{1}{3}\times d=\frac{1}{4}\times t=\frac{1}{5}\times o=\frac{1}{8}\times(o+d)$

Can you complete a similar series of equivalent expressions for the pink rod:

$p=\frac{1}{2}\times\square=\square\times(o+r)=\frac{1}{4}\times(\square+d)=\square\times 2\times o$

If we attribute numerical values to the rods, we obtain a new set of equivalent expressions.
Can you continue this sequence:

$2=\frac{1}{2}\times 4=\frac{1}{3}\times 6=\frac{1}{4}\times 8=\frac{1}{5}\times 10=\frac{1}{6}\times 12=\ldots$

Through using fractions and equivalent expressions you will see that it is possible to make a simple expression appear quite complex. For example:

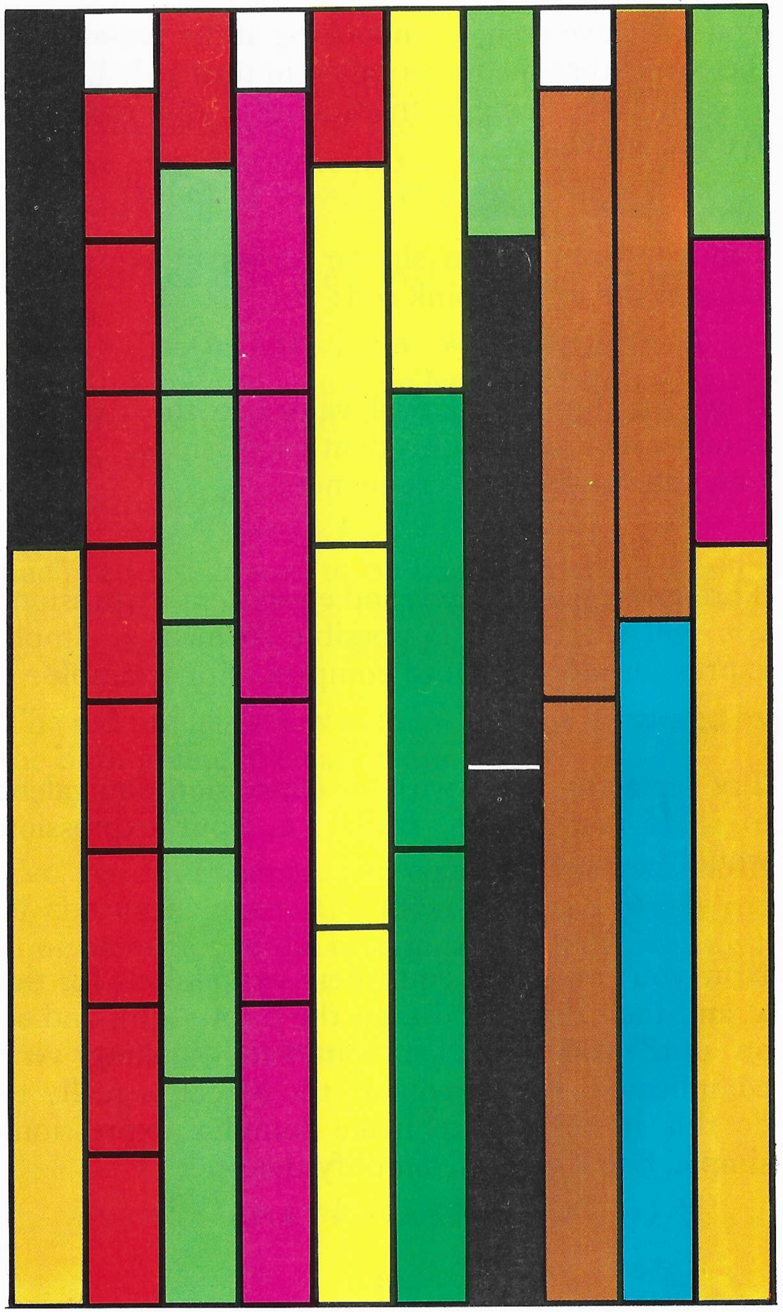
$3+4=7$

Try replacing the 3 with an expression equivalent to it. If you choose, say, $\frac{1}{2}\times 6$, your expression would become:

$(\frac{1}{2}\times 6)+4=7$

Now you could find equivalent expressions for the 4, and then the 7, and then the 6 (of $\frac{1}{2}\times 6$), and so on until you arrive at something looking very complicated. But of course, the object is really to do this in reverse, to make complex expressions simple. See if you can simplify this one:

$\frac{2}{3}\times(11-2)-\frac{4}{5}\times\frac{1}{2}\times 10=\frac{1}{8}\times 4\times 4$



The study of 17

The pattern on the opposite page represents a number of the ways in which 17 can be made. Examine it carefully. Can you answer the following questions?

How many eights are there in 17? and how much is left?

How many fives are there in 17? and how much is left?

How many twos are there in 17? and how much is left?

Is 17 odd or even?

Is 17 prime or composite?

Can you complete the following expressions:

$d+d+\square=o+b$ $\qquad$ $o+b=\square+t$

$5+5+\square=17$ $\qquad$ $\square+4+3=17$

$17-9=\square$ $\qquad$ $7+7+\square=17$

$\square=17-(4+4+4)$ $\qquad$ $5+3+\square=17$

Make up new lines for 17, and write them down, until you feel that you have discovered as much as you can about this number.

Now make the length which represents some other number of your choice, and find out as much as you can about it by making a pattern for it and writing down what you see.

Do it for 14, 18, 24 and 27.

Equivalent expressions

When we first looked at our rods, we gave to each a single name according to its colour. There were ten different rods and ten different names, one for each.

But we can find many names, or expressions, to describe a particular rod, according to its **relationship** to other rods.

Let us take, for example, the dark green rod. We can write down:

$$\begin{aligned} d &= 6\times w \\ &= 3\times r \\ &= p+r \\ &= B-g \\ &= \tfrac{1}{2}\times(o+r) \end{aligned}$$

Find as many other expressions equivalent to the dark green rod as you can. We could if we wanted keep on finding new names for it forever, without repeating ourselves.

If we choose to call the white rod 1, and use it to measure the others, we can find new names:

$$6=3+3=\tfrac{1}{2}\times 12=\ldots$$

You can extend the range of your findings further and further according to your knowledge of arithmetic. For instance:

$$\begin{aligned} 6 &= \tfrac{1}{6}\times 36 \\ &= (8\times 8)+(\tfrac{1}{5}\times\tfrac{1}{2}\times 50)-(7\times 9) \\ &= 2^2+(36\div 18) \\ &= \sqrt[2]{16}-\sqrt[3]{27}+\sqrt[2]{25} \end{aligned}$$

If we now decide to call the red rod 1 instead, we would obtain after measurement yet another set of expressions for the dark green rod:

$$3=(\tfrac{1}{4}\times 4)+(\tfrac{2}{3}\times 3)=\ldots$$

Or, with the yellow rod as the unit:

$$1\tfrac{1}{5}=2-\tfrac{4}{5}=\ldots$$

Write down several different expressions of your own for the dark green rod, like those suggested above.

Now look at the pattern on page 16. How many ways can you find of reading each of the lines?

Taking the white rod as 1 for instance, we could read the second line as:

$$\begin{aligned} 17 &= 2+2+2+2+2+2+2+2+1 \\ &= (8\times 2)+1 \\ &= 2+(\tfrac{1}{2}\times 4)+(\tfrac{1}{4}\times 8)+(\tfrac{1}{8}\times 16)+(\tfrac{1}{2}\times 16)+(9-8) \end{aligned}$$

You can read the pattern using each of the rods in turn as 1. If the light green rod were 1, the second line would read as:

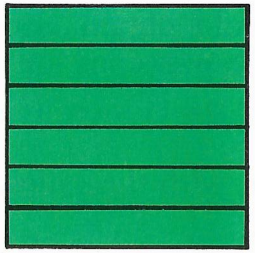
$$(8\times\tfrac{2}{3})+\tfrac{1}{3}$$

and the third line:

$$5+\tfrac{2}{3}$$

Continue until you feel you have tried all the possibilities for reading this pattern in different ways.

The binomial theorem



If you were to make three squares, the first with pink, the second with dark green and the third with orange rods, you could see that they represented

$p \times p$, $d \times d$ and $o \times o$

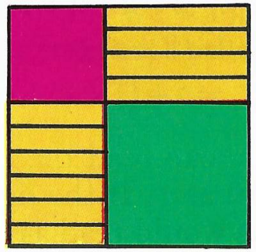
which we can write as p^2, d^2 and o^2 for short, reading the small ^{2} as **squared** to indicate what we mean.

o^2 is equivalent to $(p+d)^2$ since $o=p+d$

Can you find out which is bigger:

$(p+d)^2$ or p^2+d^2

The best way of doing this is to place the pink and the dark green squares on top of the orange one.



Clearly the orange square is bigger since the other two together cannot cover it up completely.

By placing the two smaller squares at opposite corners of the orange square, you can see that the orange area that is left uncovered is equal to two rectangles each equal to $p \times d$

In writing we put that we first found that

$(p+d)^2$ is greater than p^2+d^2

and then that

$(p+d)^2=p^2+d^2+2(p \times d)$

In the same way, try to find the relationship between the Blue square and the sum of the pink and yellow squares, remembering that $B=p+y$. Then try some examples of your own choosing.

If you measure the rods using the white as the unit, you arrive at expressions of the type:

$(3+4)^2=3^2+4^2+2(3 \times 4)$

which is equivalent to $9+16+24$ and hence to 49. $3+4=7$, and it is of course quicker simply to multiply 7 by 7 to arrive instantly at 49.

If, on the other hand, we were faced with 43^2, we would have a problem too hard to work out mentally in the normal way.

But *you* can do it in your head quite easily, with practice, if you split it up like this:

$$43^2=40^2+3^2+2(40 \times 3)$$
$$=1600+9+240$$
$$=1849$$

Try this way of working out 17^2, and 22^2, and other examples you can think up for yourself.

You have by now found out some of the surprises the Cuisenaire rods hold in store for everyone. And although this, your *First Book about the Cuisenaire Rods* has come to an end, you still have the rods, and there is still a great deal to be discovered about them. If you enjoyed reading this book and working out the problems in it, you might like to know that there are many, many other books and problem sheets also written for you that will help you explore still further.